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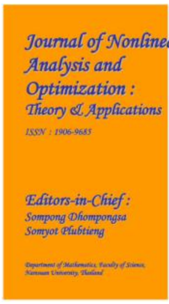

ROBUST HYBIRD FUZZY GRAPH

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ABSTRACT

Fuzzy graph theory is crucial for dealing with uncertainty in everyday situations. In this paper, we provide a simple, flexible, and fundamental fuzzy network connected with a robustness model. We tend to deal with the examination planning disadvantage presented by this strategy. Randomness and unclearness are two separate types of knowing uncertainty. This research deals with all types of uncertainty in higher cognitive processes and links Robust Hybrid Fuzzy Graph. In this paper, we introduce a type of fuzzy graphs known as Robust Hybrid Fuzzy Graph, as well as some of its properties. A Robust fuzzy network is the best tool in evidence theory for computing belief functions, plausibility functions, spanning functions, and so forth.

Keywords: Robust, Hybrid fuzzy graph, Belief measure, Plausibility measure, spanning.

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Introduction

It is quite well known that graphs are simply models of relations. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a 'Fuzzy Graph Model'.

Application of fuzzy relations area unit widespread and important; particularly within the field of clump analysis, neural networks, laptop networks, pattern recognition, decision making and professional systems. In each of those, the fundamental mathematical structure is that of a fuzzy graph.

We know that a graph is a symmetric binary relation on a nonempty set V. Similarly, a fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. The first definition of a fuzzy graph was by Kaufmann[1] in 1973, based on Zadeh's fuzzy relations [2]. But it was Goguen [3] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1968. During the same time R.T.Yeh and S.Y. Bang [4] have also introduced various connectedness concepts in fuzzy graphs.

There are few works on fuzzy graph with robustness. However, there is no work, as per our data, that has incorporated strength associate degreed fuzzy graph in an integrated degreed fuzzy graph in an integrated manner for programming drawback. Works on strength in programming drawback is found in [9], [10], [11], [12]. Some works associated with fuzzy graph coloring are there in [8], [6], [7], [5].

The paper is organized as follows. Section 2 briefly reviews some basic definitions associated with fuzzy graph. In Section 3, we describe the robust hybird fuzzy graph model. Finally, we conclude in Section 4.

PRELIMINARIES Definition 2. 1

A fuzzy graph $G = (V, \mu, \rho)$ is a non empty set V together with a pair of functions $\mu : V \rightarrow [0,1]$ and $\rho : V \times V \rightarrow [0,1]$ such that for all x, y in V, $\rho(x, y) \leq \mu(x) \wedge \mu(y)$.

We call μ the fuzzy vertex set of G and ρ, the fuzzy edge set of G respectively.

Example 2.2

Let $G = (\mu, \rho)$ be with $\mu * = \{u, v, w, x\}$.

Let

$\mu(u) = 0.7, \mu(v) = 0.8, \mu(w) = 1$, $\mu(x) = 0.5$, and $\rho(u, v) = 0.6$, $\rho(v, w) = 0.8$, $\rho(w, x) = 0.3$, $\rho(x, u) = 0.5$ and $\rho(u, w) = 0.4$.

Then G is a fuzzy graph since $\rho(u, v) \leq \mu(u) \wedge \mu(v)$ for all u,v in $\mu *$.

Definition 2.3

The fuzzy graph $H = (\nu, \tau)$ is called a partial fuzzy subgraph of $G = (\mu, \rho)$ if $\nu \subseteq \mu$ and $\tau \subseteq \rho$.

Definition 2.4

The fuzzy graph $H = (P, \nu, \tau)$ is called a fuzzy subgraph of $G = (V, \mu, \rho)$ induced by P if $P \subseteq V, \nu(x) = \mu(x)$ for all $x \in P$ and $\tau(x, y) = \rho(x, y)$ for all $x, y \in P$.

A single node is considered as a trivial path of length 0.

Definition 2.5

The strength of a path is the weight of the weakest edge of the path.

Example 2.6

Let $G = (\mu, \rho)$ be with $\rho * = \{u, v, w, x\}$. Let $\rho(u, v) = 0.2, \rho(v, w) = 0.2, \rho(w, x) = 0.3, \rho(x, u) = 0.5$ and $\rho(u, w) = 0.4$.

In G, C_1 = u, v, w, x, u is a fuzzy cycle as it contains two weakest arcs namely arcs (u,v)and (v, w) whereas C_2= u, w ,x, u is not a fuzzy cycle.

Definition 2.7

Let $G = (\mu, \rho)$ be a fuzzy graph. The strength of connectedness between two vertices x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x, y)$. An x-y path P is called a strongest x-y path if its strength equals $CONN_G(x, y)$.

Definition 2.8

An f-graph $G = (\mu, \rho)$ is connected if for every x,y in $\rho *$, $CONN_G(x, y) > 0$. If G is disconnected, maximal connected fuzzy graphs are called components. If G is connected, any two vertices are joined by a path. An arc (x,y) of a fuzzy graph G is normal if and only if $\rho(x, y) = CONN_G(x, y)$.

Definition 2.9

Complete fuzzy graph (CFG) is an f-graph $G = (\mu, \rho)$ such that $\rho(x, y) = \mu(x) \wedge \mu(y)$ for all x and y.

Definition 2.10

Evidence theory is based on two dual non additive measures: belief measures and plausibility measures.

Given a universal set X, assumed here to be finite, a belief measure is a function $Bel: (X) \rightarrow [0,1]$ such that $Bel(\phi) = 0, Bel(X) = 1$ and

$Bel(A1 \cup A2 \cup A3 \ldots \cup An) \geq \sum_j Bel(Aj) - \sum_{j<k} Bel(Aj \cap Ak) + \ldots\ldots + (-1)^{n+1} \sum_j Bel(A1 \cap A2 \cap \ldots \cap An)$

for all possible families of subsets of X.

For each $A \in (X), Bel(A)$ is interpreted as the degree of belief based on available evidence that a given element of X belongs to the set A. Belief measures are superadditive and when X is infinite, continuous from above.

Definition 2.11

A plausibility measure is a function $PI: (X) \rightarrow [0,1]$ such that $PI (\phi) = 0, PI(X) = 1$, and

$PI(A1 \cap A2 \cap A3 \ldots \cap An) \leq \sum_j P(Aj) - \sum_{j<k} PI(Aj \cup Ak) + \ldots + (-1)n + 1\sum_j PI(A1 \cup A2 \cup \ldots \cup An)$

for all possible families of subsets of X.

Definition 2.12

Belief and plausibility measures are characterized by a function $m: (X) \rightarrow [0,1]$ such that $m(\phi) = 0$ and $\sum_{A \in \mathcal{P}(X)} m(A) = 1$. This function is called basic probability assignment.

For each $A \in (X)$, the value m(A) expresses the proportion to which all available and relevant evidence supports the claim that a particular element of X, whose characterization in terms of relevant attributes is deficient , belongs to the set A.

Robust Hybird Fuzzy Graph (RHFG)

Definition 3.1

Let X is a crisp set. A robust hybird fuzzy graph is a non- empty set $V = \mathcal{P}(X) \setminus \psi$ together with a pair of functions $m : V \rightarrow [0,1]$ and $\delta: V \times V \rightarrow [0,1]$ such that for all $A, B \in V$, $(A, B) \in \delta$, whenever $A \subseteq B$ and $\delta(A, B) = m(A) \Lambda m(B)$.

Also $\sum_{A \in V} (A)$=1. RHFG can be denoted by $G = (V, m, \rho)$,where m is called the assignment function and δ is called the edge function.

Theorem 3.2

Robust hybird fuzzy graph is a fuzzy graph.

Proof

Proof is evident from the definition of robust hybird fuzzy graph.

Theorem 3.3

Number of vertices of a robust hybird fuzzy graph corresponding to a crispest X with n elements is $2^n - 1$

Proof

Let $G = (m, \delta)$be the robust hybird fuzzy graph. Then by the definition, $V = (X) \setminus \psi$ is the vertex set. So the number of vertices is $2^n - 1$.

Theorem 3.4

Number of edges of a robust hybird fuzzy graph corresponding to a crisp set with n elements is

$$(\sum_{i=1}^{n-1} (n-1)C_i)\, nC_{n-1} + (\sum_{i=1}^{n-2} (n-2)C_i)\, nC_{n-2} + \ldots\ldots\ldots\ldots\ldots\ldots + n$$

Proof

Proof is the consequence of set theory.

Let us start with singleton sets. Every vertex corresponding to singleton sets is adjacent to all the vertices corresponding to their supersets - 2-element sets, 3-element sets etc.

$$\{a\} \rightarrow \{a, b\}, \{a, b, c\}, \{a, b, c, d\} \ldots\ldots.$$

This can be done in $(\sum_{i=1}^{n-1} (n-1)C_i$ ways.

The number of singleton sets for a set with n elements is n or nC_{n-1}

So the total cases corresponding to singleton sets is

$$\left(\sum_{i=1}^{n-1} (n-1)C_i\right) nC_{n-1}$$

The vertices corresponding to 2-element sets is adjacent to all vertices corresponding to all vertices corresponding to their super sets - 3-element sets, 4-element sets etc.

$\{a, b\} \rightarrow \{a, b, -\}, \{a. b, -, -\}, \{a, b, -, -, -\}$ etc.

Theorem 3.5

Robust hybird fuzzy graph is complete

Proof

A complete fuzzy graph is a fuzzy graph $G = (V, m, \delta)$ such that $\delta(x, y) = m(x) \Lambda m(y)$

for all x and y. So by definition every robust hybird fuzzy graph is complete.

Theorem 3.6

There does not exist an edge (A,B) such that $\delta(A,B) = 1$ in a Robust Hybird fuzzy graph $G = (m, \delta)$

Proof

If possible, suppose that there exist an edge (A,B) such that $\delta(A,B) = 1$
$\Rightarrow m(A) = m(B) = 1$ by the definition of RHFG.
$\Rightarrow \sum_i (A_i) \neq 1$, a contradiction.

Definition 3.7

The fuzzy graph $H = (n, \tau)$ is called a partial Robust fuzzy subgraph of $G = (m, \delta)$ if $n(A) \leq m(A)$ for all A and $\tau(A,B) \leq \delta(A,B)$ for all A ,B such that $A \subseteq B$

The fuzzy graph $H = (P, n, \tau)$ is called a robust fuzzy subgraph of $G = (V, m, \delta)$ induced by P if $P \subseteq V$, $n(A) = m(A), \tau(A,B) = \delta(A,B)$ for all A , B such that $A \subseteq B$.

Proposition 3.8

The partial Robust fuzzy subgraph and Robust fuzzy subgraph of an RHFG need not be a RHFG

Proof

For a partial robust hybird fuzzy graph and robust hybird fuzzy graph, $\sum_i (A_i)$ equal to 1.
But $\sum_i (A_i) \leq 1$.

Theorem 3.9

The partial robust fuzzy subgraph of an RHFG is an RHFG if and only if $m(A) = n(A)$ for all A and $\tau(A,B) = \delta(A,B)$ for all A, B.

Proof

Let G be a RHFG .
By definition $\sum_i (A_i) = 1$.
Let $H = (n, \tau)$ be a partial robust fuzzy subgraph of G.
For a partial Robust fuzzy subgraph H= (n, τ) of $G = (m, \delta)$, $n(A) \leq m(A)$ for all A and $\tau(A,B) \leq \delta(A,B)$.
But $\sum_i n(A_i) \neq 1$ if $n(A) < m(A)$.So $n(A) = m(A)$ for all A which implies $\tau(A,B) = \delta(A,B)$ for all A, B.
Converse is obvious.

Definition 3.10

The vertex in a robust hybird fuzzy graph which is adjacent from every other vertex is called complete vertex.

Definition 3.11

In an RHFG $G = (m, \delta)$, a path P of length n is a sequence of distinct vertices $A0, A1, \ldots\ldots An$ such that $\delta(Ai - 1, Ai) > 0, Ai - 1 \subseteq Ai$, $i = 1,2, \ldots \ldots n$ and the degree of membership of the weakest arc is its strength.

Theorem 3.12

Maximum length of a path P in a RHFG with n vertices is n-1.

Proof

Consider a RHFG with n vertices $A0, A1, . . An$. Start from an arbitrary vertex Ai. Since there are only $n - 1$ vertices remaining, choose a vertex Aj such that $\rho(Ai, Aj) > 0$, $Ai \subseteq Aj, i \neq j$.Similarly choose a vertex Ar from the remaining $n - 2$ such that $\rho(Aj, Ar) > 0, Aj \subseteq Ar$, and so on. Since there are only n distinct vertices the process must terminate at a vertex Ap such that,

$$\rho(Ap - 1, Ap) > 0, Ap - 1 \subseteq Ap, p < n.$$

We get the sequence $Ai, Aj, - - - - Ap$ which is of length less than n and equal to $n - 1$ only if every vertex is ordered by the relation $\subseteq$.

Theorem 3.13

RHFG does not contain cycles and so fuzzy cycles.

Proof

Since in a RHFG $G = (V, m, \delta)$, for all $A, B \in V$, $(A, B) \in \delta$ whenever $A \subseteq B$ there will not be an edge (B,A) and so a cycle.

Definition 3.14

The vertex A such that $m(A) \leq m(B)$ for any B is called arc is called weakest vertex. The arc (A,B) determined by the weakest vertex is called weakest arc.

An RHFG can have more than one weakest vertices and weakest arcs

Definition 3.15

A robust hybird fuzzy graph $G = (V, m, \rho)$ is evidently connected if for every A, B in V with $A \subseteq B, CONNG\ (A, B) > 0$.

Two vertices A and B such that $A \subseteq B$ or $B \subseteq A$ are evidently connected if there exist an edge (A,B) or (B,A) .

Proposition 3.16

RHFG is always disconnected

Proof

For $X = \{x, y\}$ there does not exist a path between x and y.

Definition 3.17

Consider the Robust fuzzy matrix $MG = (mAB)$ of $G = (m, \delta)$, where $mAB = m(A, B)$ $if\ A \subseteq B\ and$ 0 $otherwise$.

The matrix M_G^k such that $M_G^k = M_G^{k+1}$ where k is a positive integer is called the evidence reachability matrix of G denoted by $R_G = (r_{AB})$

Definition 3.18

Two vertices A and B of a robust hybird fuzzy graph is said to be mutually disconnected if there is neither an edge (A,B) nor (B,A).

The vertices A and B are mutually disconnected.

Proposition 3.19

Belief and plausibility measures of a complete vertex are always one.

Conclusion

In this paper, we create a brand-new type of fuzzy network known as a robust hybird fuzzy graph. Completeness, routes, connectivity, and other traits are among them. We also explore how fuzzy graphs may be used in robust theory to determine belief measures, plausibility measures, and other parameters under uncertain conditions. We can calculate belief measure using RHFG as $Bel(A) = m(A) + \sum B\ m(B), (B, A)$ is an edge; where $m(A)$ is the degree of the vertex A in RHFG.

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